3. INDUSTRIAL COMPUTATIONAL MECHANICS

3.1 Computational Structural Mechanics

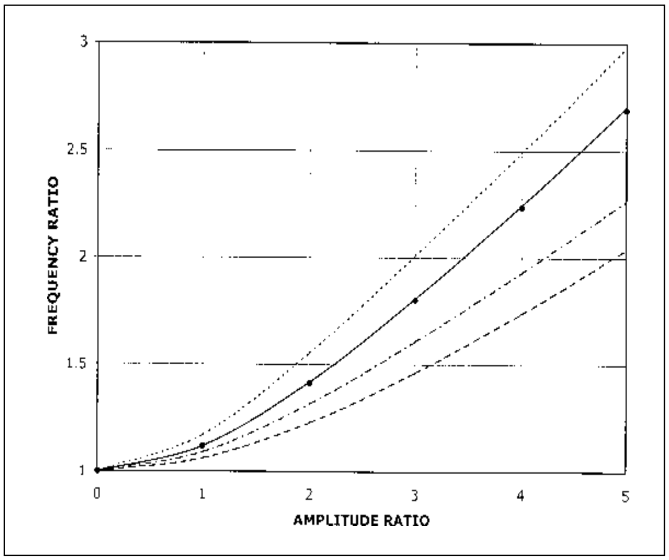
3.1.1 Modelling of Nonlinear Dynamics

Nonlinear vibration problems of beams and plates have been approached using finite element modelling for quite some time now. The nonlinear stiffness terms are understandably difficult to represent and extra-variational simplifications are often resorted to. Two such simplifications are found frequently in the literature because they considerably simplify the computation of the non-linear stiffness terms from the non-linear strain energy terms. A quasi-linearisation approach allows a simple representation of the non-linear terms. Alternatively, the neglect of inplane deformation terms allows the stiffness matrix to be formulated in terms of transverse deflections and section rotation terms alone.

Two different finite element models are developed, incorporating all possible combinations of the extra-variational simplifications for various commonly found boundary conditions. For each element, four versions appear -

VC - Variationally correct model with no simplification
VC* - Variationally correct, but inplane deformation neglected
QL - Quasi-linearised model, where nonlinear strain is linearised
QL* - Quasi-linearised model, but inplane deformation neglected

Fig. 3.1.1.1. Evaluation of finite element models for simple-supported beam



Numerical computations are performed systematically for all versions of both elements. Results show that quasi-linearisation reduces stiffness considerably, while neglect of inplane displacement terms registers excessive nonlinear stiffness. When both simplifications are introduced together (QL*), there is a fortuitous cancellation of errors; Fig. 3.1.1.1 shows a graphical representation of how the frequency ratio varies with amplitude of vibration for the various versions of a finite element model for the nonlinear vibration of a simple-supported beam.

(G Prathap and S R Marur; *CSS Foundation, Madurai)*

3.1.2 Modelling of Smart Structures

Piezoelectric materials have tremendous potential in vibration control applications as distributed sensors and actuators. These can be either surface bonded or embedded into composite structures. The modelling of and for active vibration control has become a very important area in aerospace applications. In this work, the active vibration control of composite sandwich beams with distributed piezoelectric extension-bending and shear actuators is taken up. The quasistatic equations of piezoelectricity have been employed to derive a finite element model capable of modelling two different kinds of piezoelastically-induced actuation in an adaptive composite sandwich beam. The piezoelastic constants couple with transverse electric field to develop extension-bending and shear-induced actuation.

Numerical experiments show that the shear actuators induce distributed forces/moments in a composite sandwich beam in contrast to the extension-bending actuators, which develop only concentrated forces/moments. A modal control model is developed based on Linear Quadratic Regulator which, in turn, is based on Independent Modal Space Control approach (LQRC/IMSC). The modal gain of each mode is computed and used to estimate the active stiffness and the active damping introduced by both actuation mechanisms. Dynamic response studies were conducted to assess the performance of each type of actuation in reducing the disturbance due to a sinusoidal force. It was confirmed that the shear actuator was more efficient than the extension-bending actuator in actively controlling the vibration. Figure 3.1.2.1 shows the first mode control with shear actuation and extension-bending actuation for a composite sandwich

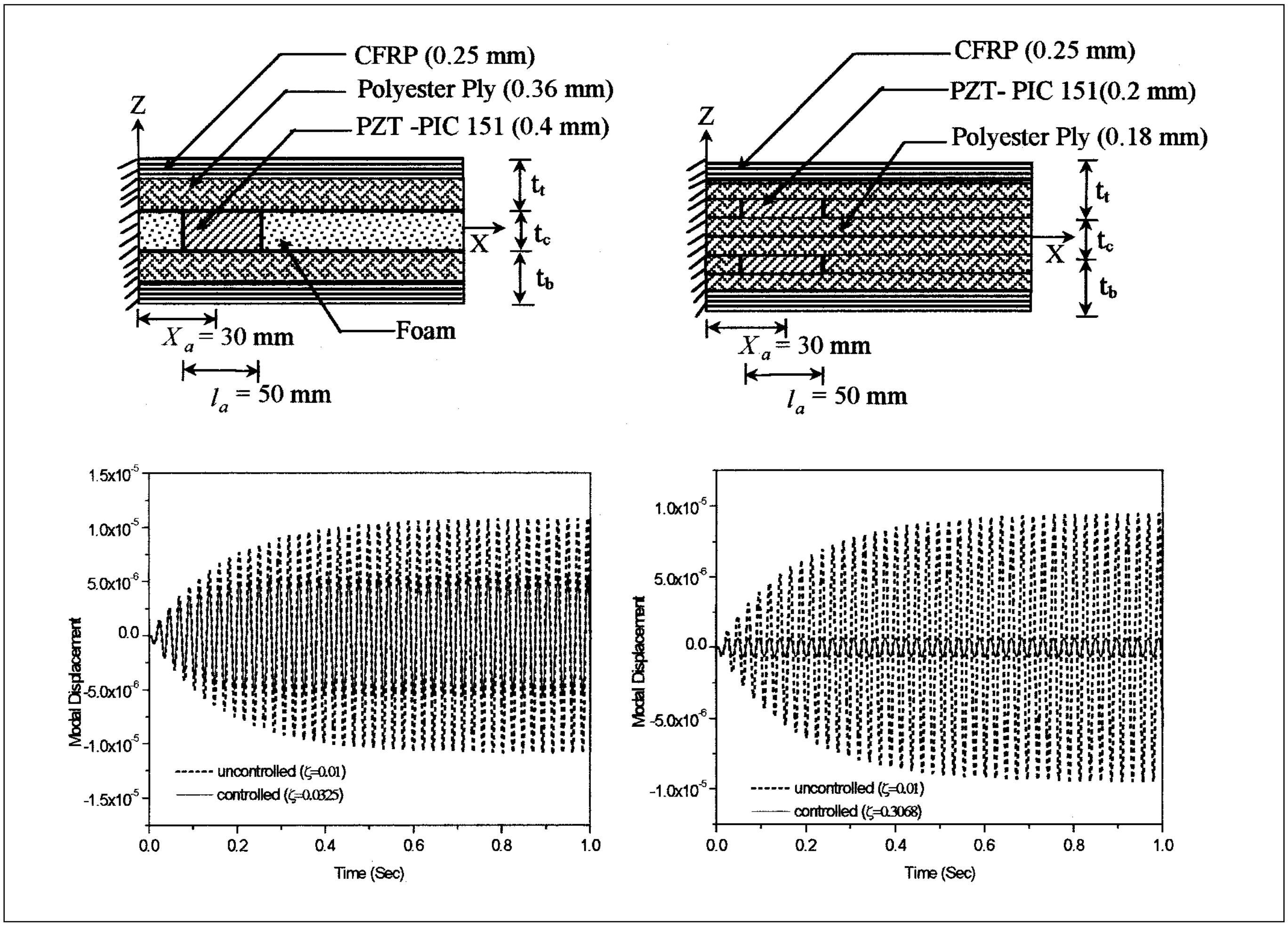


Fig. 3.1.2.1. Composite sandwich beams first mode control with shear actuation and extension-bending actuation (former is shown in the left panels and the latter in the right); top panels show the configurations and the bottom panels the establishment of control.

beam (configuration shown in top panels) and the bottom panels indicate that the shear actuators are more efficient than the extension-bending actuators in controlling the first elastic mode.

A four-node piezoelastic field-consistent plate element based on the Mindlin-Reissner theory was developed to study the influence of one and two dimensional piezoelectric actuation on the active vibration control of smart panels. Numerical experiments investigated the influence of uniaxially polarised and biaxially polarised surface bonded piezoelectric actuators on the static, dynamic and control characteristics of aluminium panels with frequently encountered boundary conditions. The uniaxially polarised actuator is found to be more efficient in controlling the panel vibration than the biaxially polarised one. Figure 3.1.2.2 shows the 1-D and 2-D piezoelectric influence on active damping of smart panels for the clamped (C-C-C-C) and cantilever (C-F-F-F) planforms.

(S Raja and G Prathap; *NAL, Bangalore)*

3.1.3 Error Analysis

The finite element method has proved itself to be a powerful tool for generating approximate solutions of differential equations by employing the principles of variational calculus. Though the method originated from the principles of structural mechanics, it is now considered as a general tool to handle the varied types of differential equations of mathematical physics. In principle, the method, as opposed to the infinitesimal calculus, is designed to approximate the exact solution of a differential equation in discrete, piecewise domains, so that the infinitesimal element of calculus is approached by convergence of results with decreasing element size and consequently increasing discretization density. Error analysis of such methods is an important exercise to ensure the quality of these approximate solutions.

It has been recently established (the stress correspondence paradigm of Prathap) that finite element analysis is effectively a process of finding the best-fit solutions to the

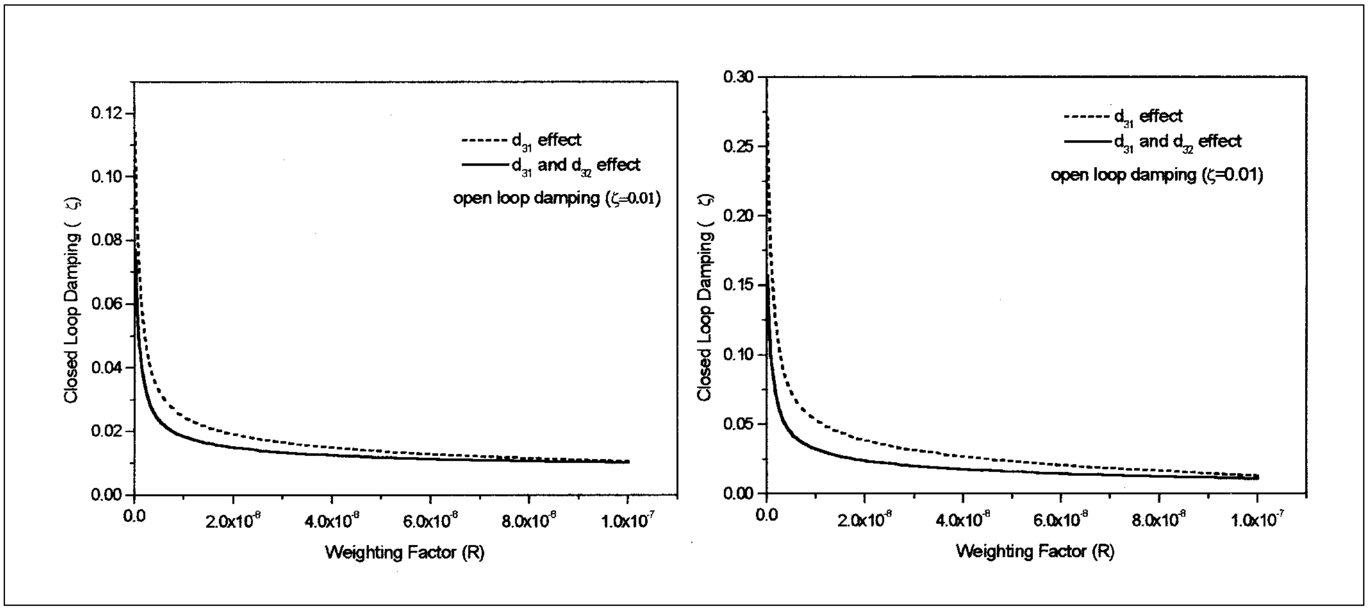


Fig. 3.1.2.2. 1-D and 2-D piezoelectric influence on active damping of smart panels (C-C-C-C; C-F-F-F)

analytical solutions of differential equations. In computational structural mechanics, by best-fit, we mean that the finite element procedure computes strains or stresses which are least-squares approximations of the actual state of strain or stress. This can be derived as an orthogonality condition from the Hu-Washizu principle. Such a principle also emerges from the projection theorems of functional analysis, which show how the analytical solutions are projected onto the approximate solution subspaces in the Hilbert space. Interestingly, one often encounters problems where this rule is seemingly violated. Work was carried out to study this carefully. Studies with a model boundary value problem with Dirichlet conditions (the axi-symmetric Laplace equation corresponding to an electromagnetic flux problem) show that the best-fit interpretation of strain and stress recovery is valid if we take into account the spurious stiffening effect produced in the assembled discretised model. A priori error models were developed to predict that the actual computation using assembled finite element equations would reflect an artificial stiffening, which was confirmed during the numerical experiments. An interesting analogy of the electromagnetic flux problem with the structural problem was seen. The stiffening effect is attributed to an enhanced or distorted flux, in analogy with the spurious enhanced support reactions seen in finite element models in structural mechanics. Both the spurious and true support reactions or fluxes satisfy overall equilibrium conditions.

(S Mukherjee and G Prathap; *NAL, Bangalore)*

3.2 Equivalent Continuum Analysis of Jointed Rock

A simple practical method has been developed to characterize the strength and stiffness of jointed rock masses. Empirical relations for the strength and stiffness of rock masses have been obtained based on the statistical analysis of a large amount of experimental data. These relations were used for developing a representation of the jointed rock mass as an equivalent continuum; the effect of joints in the rock mass is taken into account through a joint factor. The equivalent continuum model has been validated against experimental results for jointed rock masses with different joint fabrics and joint orientations and also with the results from explicit modeling of joints using FEM. In the equivalent continuum approach, the discontinuous rock body is modeled as an equivalent continuum, the properties of each element defined in terms of some combination of the properties of the intact rock and those of the joints. In the case of explicit modeling, the joints are explicitly represented by a joint element. Comparison of the equivalent continuum results and the explicit modeling results for a multiply jointed specimen of Agra sandstone is given in Fig. 3.2.1.

The developed model has also been applied to calculate the deformation around a large power station cavern in rhyloite rock at 200 m depth. The cavern measured 28 m in width, 51 m in height and 161 m in length. The amount of jointed rock mass excavated due to opening of the cavern was estimated to be about 1.9 x10^5 m^3 of rock mass. Figure

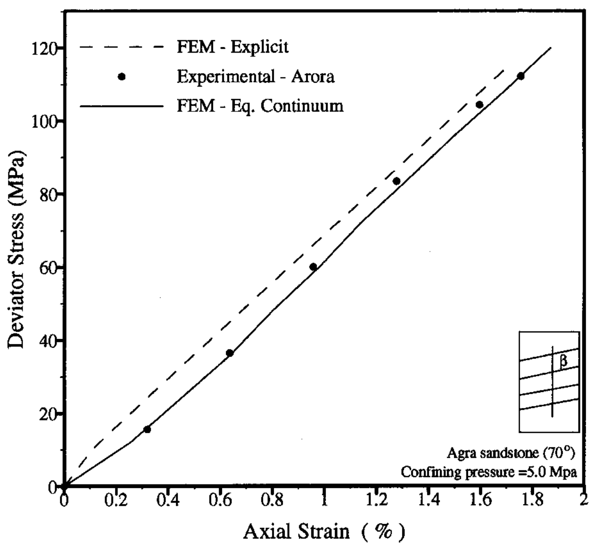


Fig. 3.2.1. Comparison of FEM results where joints in a multiply jointed specimen of Agra sandstone are explicitly represented and where they have been approximated through an equivalent continuum model; experimental results are also provided.

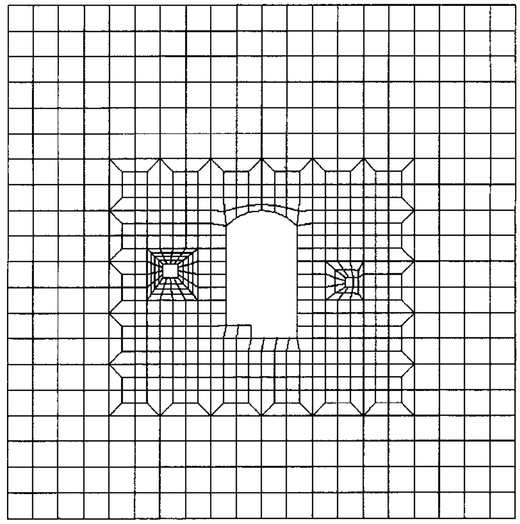


Fig. 3.2.2. Finite element mesh used for modelling a cavern and the surrounding rock.

3.2.2 shows the finite element mesh for the cavern and the surrounding rock. Equivalent continuum analysis of cavern excavation gives the relative displacement values slightly lower than the observed values of relative displacement in the field. The observed values of relative displacement reduce sharply at a distance of 5 m to 20 m from the cavern wall; the values of relative displacement obtained using equivalent continuum analysis reduce gradually at points away from the cavern wall. Overall, the results match well within the limits of numerical and experimental framework.

Equivalent continuum model is able to give a fair estimate of the rock mass behavior. The only input data required for the analysis is the properties of intact rock and the joint properties for estimating the joint factor. Shiobara power cavern analysis using equivalent continuum approach shows that the field problem analysis is simplified to a large extent while, at the same time, giving a fair estimate of the behavior of the surrounding jointed rock with minimum input data.

(Sridevi Jade and T G Sitharam; *IISc, Bangalore)*

3.3 Moving Grid Methods for Bioremediation

Previous work on bioremediation of the soil system at Borhola Oil fields had shown that the bioremediation process is a moving front that advances from the macropores into the micropores of the soil aggregrates along with the oxygen front. It was surmised that the model itself can be made more efficient by converting it to a moving boundary problem and employing adaptive moving grid methods to track the moving front.

Moving grid methods are becoming increasingly popular for solving several kinds of parabolic and hyperbolic partial differential equations (PDEs) involving fine scale structures such as steep moving fronts, emerging steep layers, pulses, shocks etc. Moving grid methods use nonuniform space grids and move the grid continuously in the space-time domain.

A literature survey of available moving grid methods was carried out. Three different moving grid methods were identified for experimentation. The first method is a finite difference method based on the Lagrangian description of the PDEs and a smoothed equidistribution principle to define the grid positions at each time; this is coupled to a spatial discretization method which automatically discretizes the spatial part of the user-defined PDEs following the Method of Lines approach.

The second method is a gradient weighted moving finite element method (GWMFE). The gradient weighting amounts to the use of weighting functions in the finite element

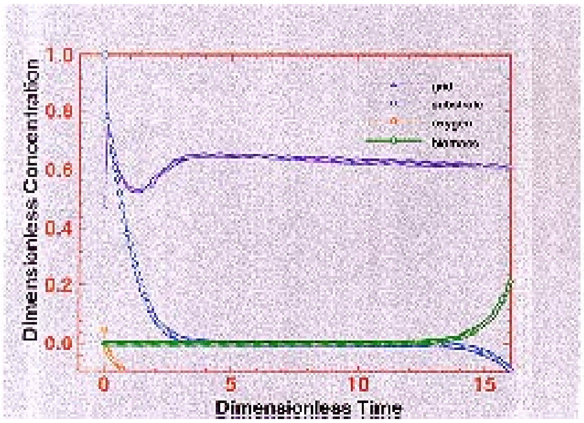


Fig. 3.3.1. Evolution of concentrations during remediation process computed using a finite difference moving grid method based on method of lines.

formulation that depend on the gradient u_x of the solution. GWMFE formulation is done by calculating the inner products of the PDEs with piecewise linear basis and test functions on a nonuniform grid. The GWMFE equations are a discretized approximation to the force balance equation; the right hand side represents the applied forces on the manifold from the PDEs and the left hand side represents viscous drag forces opposing the normal motion of the manifold. The GWMFE manifold moves its nodes in such a way that the forces when concentrated onto the nodes, exactly balance at each node. The treatment results in a more robust process in which the parameter tuning of the weighted functions becomes easier and also less critical.

The third method is an Adaptive Mesh Refinement technique (AMR). The AMR approach refines in time as well as space. The finer grids are placed adaptively in the subregions that require a better resolution over the coarser grid covering the region. The AMR method is implemented recursively to the finest level. If the refinement factor between a finer level and the next coarser level is r, then the grids on the finer level will be advanced r time steps for each coarser time step. The regridding of the next level includes computing the physical locations for each fine grid and copying or injecting the solution from the old grid to the new grid by interpolation. The output order forms a *W-cycle* since the method replaces the coarse grid with its immediate finer grids if it exists and also the method is done recursively for all the grids; each of the subgrids is referred to as a patch.

The first method was applied to our bioremediation problem and the results obtained are as shown in Fig. 3.3.1. The results showed some deviation with the plots obtained using the fixed grid method. Further studies have to be made to determine the reasons for deviation when adapted to a moving-grid interface. However the method showed promise as the figure clearly shows the reduction of contaminant concentration with the increase in biomass concentration and, more importantly, application of moving-grid interface to the bioremediation problem has significantly reduced the computation time by several orders of magnitude.

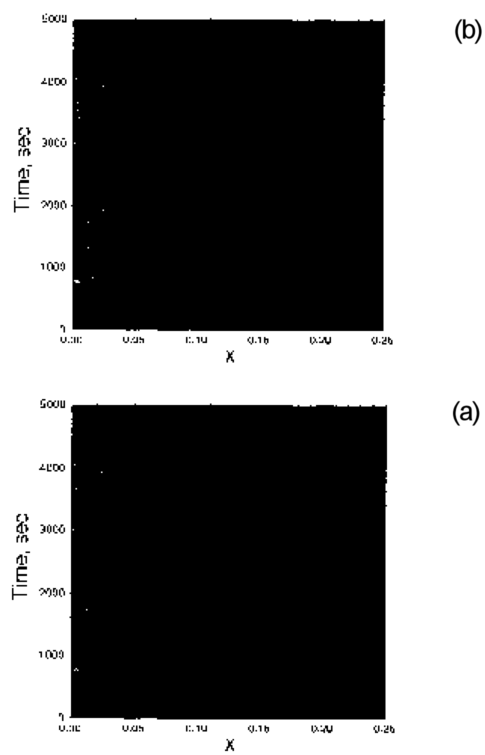
The implementation of the other two moving grid methods in the bioremediation problem are being studied and the results will be reported later.

(A N Navaneeth and T R Krishna Mohan)

3.4 Oscillatory Rayleigh-Bénard Convection in a Viscoelastic Liquid

Convective motion that arises when a thin layer of viscoelastic fluid is heated from below (Rayleigh-Bénard convection) is being investigated; we use Oldroyd B constitutive equations to model the fluid. Besides the

Fig. 3.4.1. Space-time plot of v showing development of instability; (b) is a continuation of (a) in time.



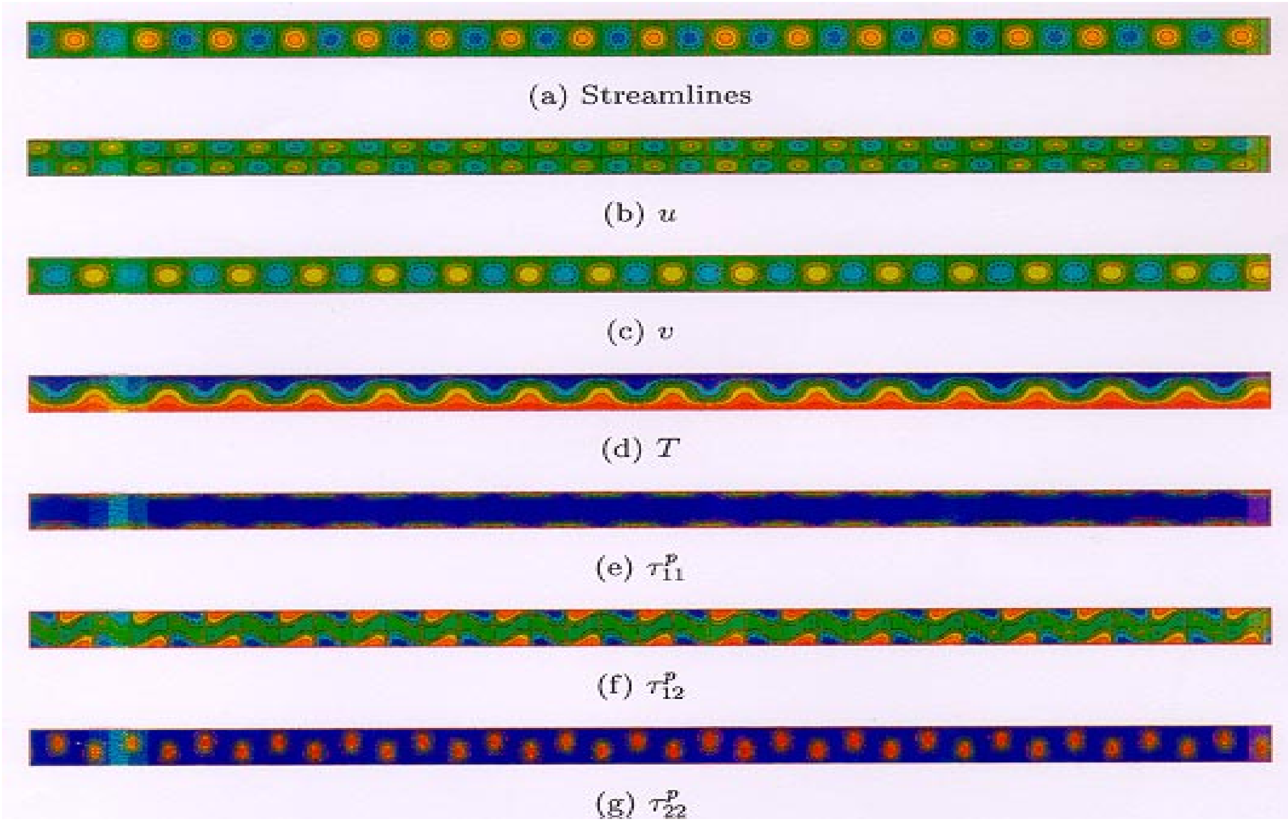


Fig. 3.4.2. Streamlines are shown in the top panel and the subsequent panels show contours of the various indicated quantities at t= 9500s.

usual stationary convection that is observed in a Newtonian fluid, it is now known that an oscillatory convection can also be obtained in viscoelastic fluids for certain values of the parameters. Unlike the Newtonian fluids, there are two competing time scales in viscoclastic fluids which make this possible. Back of the envelope calculations had indicated that the relaxation time required for oscillatory convection to appear in viscoelastic fluids is typically of the order of 1s, which is quite large compared to the then known values for these fluids. It was therefore believed that oscillatory Rayleigh-Bénard convection cannot be observed in viscoelastic fluids in realistic experimental conditions. However, large values of the relaxation time were recently measured for DNA suspensions, and, subsequently, oscillatory Rayleigh-Bénard convection was also observed in these suspensions in an annular geometry.

Simulations of the viscoelastic Rayleigh-Benard convection has been carried out at C-MMACS to describe the development of the instability and the resulting oscillatory motion within the convective cells. Figure 3.4.1 shows the development of the instability and Fig. 3.4.2 shows the convective flow between the plates at a certain instant of time.

(A Kumar)