3.1 Modelling of Smart Structural Systems

(a) Piezo-Hygrothermo-Elastic Analysis of Laminated Flat and Curved Plates With Active Controls

The smart materials possess multi-functional capabilities such as load carrying, sensing, and actuation, that can be simulated properly through mathematical modelling to realise real time smart/adaptive structural systems. Incorporation of advanced multi-functional material systems such as piezoelectric materials enhances the structural performance through active effects (active stiffening, active damping) by minimising the vibration amplitude or other undesired effects. The composite materials may be exposed to moisture and temperature during their service life (high speed aircraft, rocket, launch vehicles, etc). And the thermal stresses due to aerodynamic heating also may lead to buckling and dynamic instability of the structures. Therefore, the behaviour of the piezoelectric composite structures may be affected under hygrothermal environment. The presence of hygrothermal strain not only modifies the stiffness of the piezoelectric laminates but also influences the actuation and sensing behaviour of piezoelectric lamina resulting in different control performance.

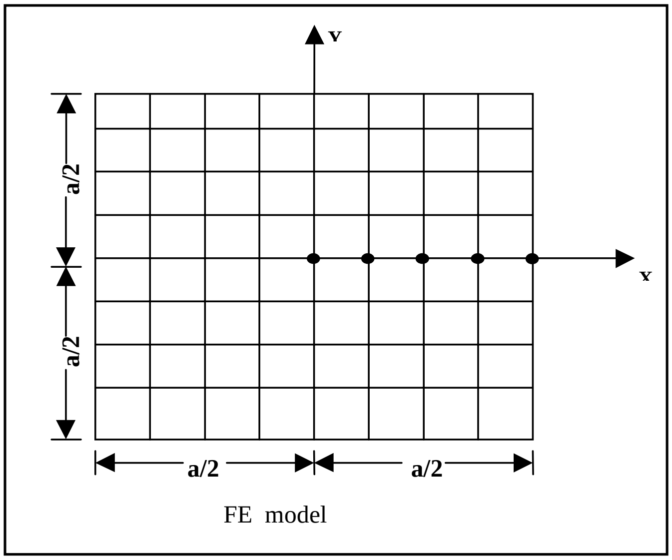

Fig. 3.1 Simply supported square laminated plate: a/h = 100. Materials: CFRP/PZT.

With the development of active fibre composites, the realisation of adaptive structural system has become a reality. However, modelling the anisotropic actuation ($d_{31} \neq d_{32}$) induced by the piezocomposite ply has been an area of research only in recent years. The characteristics of piezocomposite ply may be influenced by hygrothermal strain because the piezo fibres are embedded in polymer matrix. Therefore, it is necessary to study the hygrothermal effect on the piezoelectric anisotropic actuation behaviour of smart laminated composites. In addition, some percentage of piezoelectric anisotropy can also be introduced by directionally attaching the monolithic piezo crystal ($d_{31}=d_{32}$) on a substrate through minimizing the effect of d_{32} actuation.

A finite element procedure is developed to model the coupled piezoelectric anisotropic behaviour ($d_{32}/d_{31}=\psi$) with hygrothermal strain field using 9-node plate element (Fig. 3.1). Further, the influence of hygrothermal strain on anisotropic actuation and sensory behaviour is studied. Piezoelectric anisotropy significantly influences the control behaviour both actuation as well as sensing.

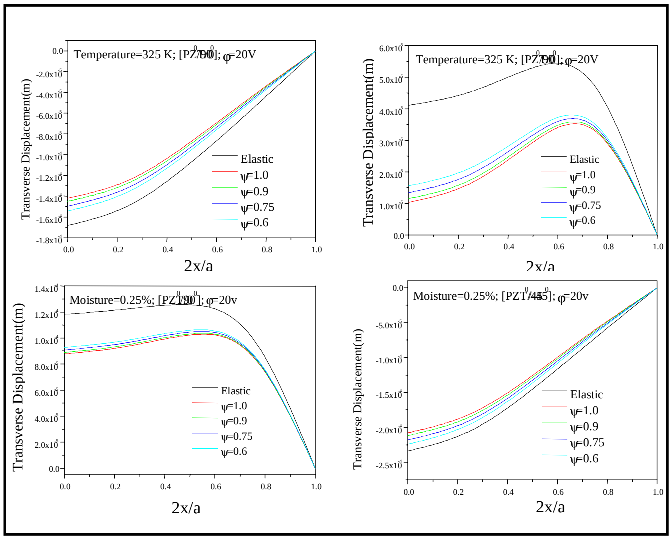

Fig. 3.2 Influence of piezoelectric anisotropy on actuation behaviour of laminated plates.

(b) Experimental Validation of FE and Control Models

The application of an active control system is quite involved in terms of modelling the structural system, design of control law, implementation of control scheme etc. A multi-input and multi-output feedback control system is designed and tested to demonstrate the vibration suppression on sandwich, laminated beams, and plate structures.

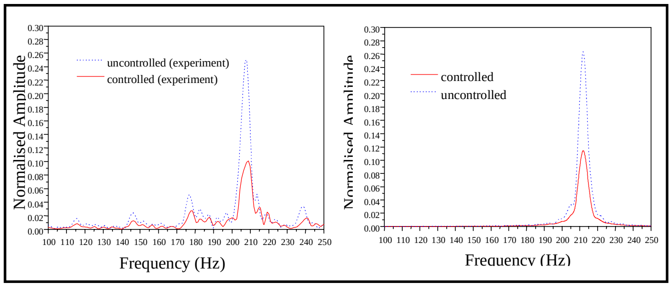

Fig. 3.3 Sandwich beam (CFRP/AL honeycomb/CFRP) first mode control: Simulated and experimental results.

Active stiffening and active damping effects in reducing the vibration amplitude are experimentally studied. A typical result is given to show the mathematical simulation and experimental validation for a sandwich beam vibration control.

(S Raja and G Prathap)*
*(*NAL, Bangalore)*

3.2 Error Analysis in Finite Element Method

Considerable efforts are being made in recent years to automate finite element methodology. For this, rational formulation of a priori error estimates is desirable. In this direction, studies were continued on the use of the Projection Theorem in Function spaces to interpret delayed convergence in the field inconsistent three-noded isoparametric Timoshenko beam elements. Interestingly, it has been shown that both field consistent and field inconsistent formulations yield results satisfying the projection theorem in an element

Energy of the strain error = Error of the energies

This is a fundamental feature of all finite element solutions. By comparison of results, it has been demonstrated that while the field-inconsistent two-noded linear Timoshenko beam element locks severely, the three noded element shows mild locking and delayed convergence, and shows similar behavior as the field-consistent two-noded beam element for thin beams. In fact, in contrast to the field-inconsistent two-noded element, the field-inconsistent three-noded element performs *better* as the beam gets *thinner*. Explicit expressions of the strains and error norms in elements have been derived and confirmed through numerical values from FEM computer codes.

Earlier work has proved with complete rigor that finite element analysis is effectively a process of finding the “best-fit” solutions to the analytical solutions of differential equations. In computational structural mechanics, by “best-fit”, we mean that the finite element procedure computes strains or stresses which are least-squares approximation of the actual state of strain or stress. This can be derived as an orthogonality condition from the Hu-Washizu principle. Such an interpretation also emerges from the projection theorems of functional analysis, which show how the analytical solutions are projected onto the approximate solution subspaces in the Hilbert space.

In a problem where Dirichlet constraints were used at both ends in the approximate solution of the Laplace equation, it appeared that the best-fit rule was violated. Careful examination has shown that the best-fit interpretation of strain and stress recovery through finite element analysis for a boundary value Dirichlet problem is valid if we take into account the spurious stiffening effect in the element, that arises from the error in the boundary flux from Dirichlet constraints only. In other words, the FEM solution with Dirichlet boundary conditions only (DDFEM) corresponding to a given analytical solution is actually the best-fit to a stiffened analytical solution. In contrast, the FEM solution with Dirichlet and flux boundary conditions (DFFEM) is the best-fit to the original analytical solution. The electrostatic problem of the charge free domain satisfying Laplace equation is taken as an illustration.

A priori estimation of errors in flux resulting from the Dirichlet constraints at the continuum boundaries are made and subsequently an interesting criterion for mesh optimization is proposed.

Studies so far on deriving error estimates from beams, and plates modelling beam-like situations show that optimal solutions are obtained when an entropy measure is minimised. In all cases, the best-fit rule of stress correspondence is satisfied, and at the optimal solution, a global entropy of energy error is minimised. In many cases, this coincides with minimum error of stress discontinuity at connecting nodes or edges.

(S Mukherjee and G Prathap)*
*(*NAL, Bangalore)*

3.3 Computational Modelling and the Logic of Discovery

Computational Modelling offers a unique laboratory to experiment with the various problems in philosophy of science, to study scientific reasoning, and particularly, on whether a logic of discovery exists or not, and whether this requires a logic larger than the formal logic embodied by the induction and deduction approaches.

Our work on formulating error estimates for finite element computations by identifying first principles, allows us to witness at first hand, this process of what we may call the Logic of Discovery. Both the Piercian and Popperian paradigms are seen to be complementary in this interpretation. First, singular and surprising patterns are noticed and stated as empirical laws (induction-Pierce; problem-Popper). Next, scientific hypotheses are projected as first principles (abduction-Pierce; observe carefully and propose bold conjectures-Popper).

Finally, these are then evaluated and tested through a formal logical process involving analytical predictions from the first principles yielding theoretical laws (deduction-Pierce; predict-Popper) and evaluating these theoretical laws against the empirical laws identified earlier (induction-Pierce; experiment and refute-Popper). If the conjectures fail to survive this reality check, one repeats the cycle again.

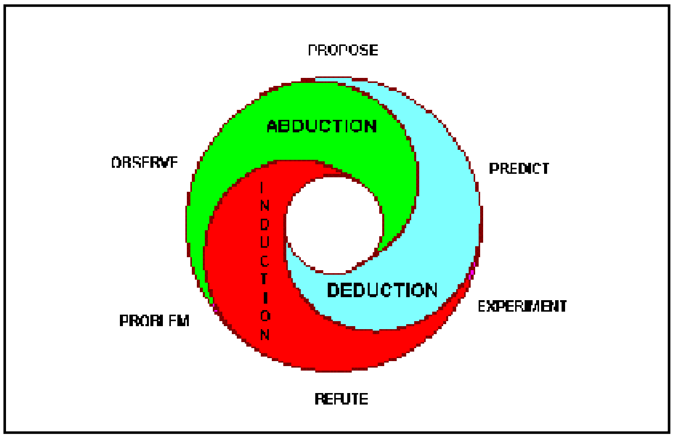

Fig. 3.4 The logic of discovery - Popperian concept.

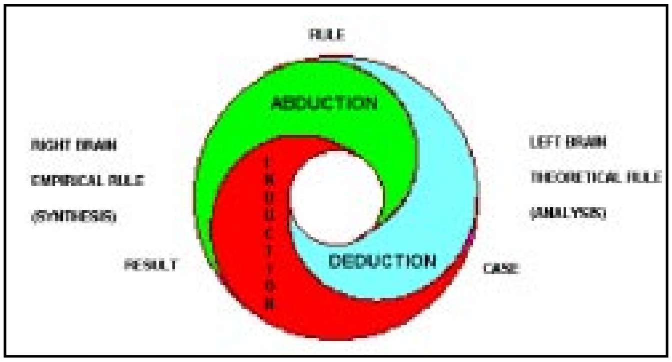

Fig. 3.5 The logic of discovery - Piercian concept.

Popper was pessimistic that a formal Logic of Discovery could be found. Recent research in Computational Philosophy of Science (an experimental epistemology) and the use of Artificial Intellegence (AI) in Exploratory Data Analysis and heuristic hypotheses formation have been greatly inspired by the Piercian three stage process of induction, abduction and deduction. Here, the larger process of science formation is seen to comprise a formal Logic of Justification based on deduction and induction and a less formal Logic of Discovery based on abduction. Our work on setting up of error estimates for finite element computation (an unsolved problem) by identifying the stress correspondence principle from a priori concepts shows how the scientific reasoning process works. A general representation of this as a Wheel of Truth (a Wheel of Dharma or the Dharmachakra?) is shown in Piercian and Popperian terms in Figures 3.4 and 3.5.

(G Prathap)

3.4 Characterization of Strength and Deformation of Jointed Rock Mass

To model the highly complex behavior of jointed rock masses, the strength and deformability of jointed rock masses should be expressed as a function of joint orientation, joint size and frequency. Moreover it is not possible to represent each and every joint individually in a constitutive model. Thus there is a need for a simple equations/relationships which can capture reasonably the behavior of jointed rock mass using minimum input. Statistical analysis of large amount of experimental data of jointed rock masses from the literature has been compiled and used for characterization of strength and deformation of jointed rock. The method used recognizes that the rock will act both as an elastic material and a discontinuous mass. Considering the inherently inhomogeneous nature of rock masses, this approach attempts to obtain simple statistical relationships to represent the properties of jointed rocks.

An effort has been made to arrive at empirical relationships which express the strength and deformation of jointed rock as a function of intact rock properties and a joint factor. Large amount of experimental data of uniaxial compressive strength ratio and elastic modulus ratio versus joint factor of the jointed rock specimens was digitally filtered to reduce the scatter in the data. Linear and nonlinear relationships between the uniaxial compressive strength, tangent elastic modulus at different confinements and joint factor have been arrived at by using least square fitting for linear relationships and Lorentzian minimization for nonlinear relationships. Least square minimization assumes that the x values are accurately determined and that an error exists only in the dependant variable y. The errors are assumed to map a Gaussian profile and are normally distributed. Lorenzian minimization is very robust when the data is noisy and also converges quite rapidly. Based on the above analysis, uniaxial compressive strength and elastic modulus obtained from uniaxial compressive tests and triaxial tests of jointed rock at different confining pressures are expressed as a function of the joint factor and intact rock properties. The joint factor depends on joint orientation, joint frequency and joint strength. Hence, knowing the intact rock properties and the joint factor, the jointed rock properties can be reliably estimated.

(Sridevi Jade)